Current and Resistance
Current

• In our previous discussion all of the charges that were encountered were stationary, not moving.
• If the charges have a velocity relative to some reference frame then we have a current of charge.
Current

• Definition of current:
Note

• The current may or may not be a function of time.
• If a battery is initially hooked up to a loop of wire there is a potential difference between on end of the wire and the other, therefore, the charges in the wire will begin to move.
• Once equilibrium is reached the amount of charge passing a given point will be constant.
• However, before equilibrium the current will be increasing and therefore it will be a function of time.
Average Electric Current

- Assume charges are moving perpendicular to a surface of area $A$.
- If $\Delta Q$ is the amount of charge that passes through $A$ in time $\Delta t$, then the average current is

$$I_{av} = \frac{\Delta Q}{\Delta t}$$
Instantaneous Electric Current

• If the rate at which the charge flows varies with time, the instantaneous current, $I$, can be found
Direction of Current

- The charges passing through the area could be positive or negative or both
- It is conventional to assign to the current the same direction as the flow of positive charges
- The direction of current flow is opposite the direction of the flow of electrons
- It is common to refer to any moving charge as a charge carrier
Current Density

We can define the current density as the current per unit area through a surface. The current can now be expressed as:
Current Density

• Here $d\mathbf{A}$ is a vector that is perpendicular to the differential surface area $d\mathbf{A}$.
• If the current is uniform across the surface and parallel to $d\mathbf{A}$ then we can write:
Example

- The Los Alamos Meson Physics Facility accelerator has a maximum average proton current of 1.0 mA at an energy of 800 Mev.
Example cont.

• a) How many protons per second strike a target exposed to this beam if the beam is of circular cross section with a diameter of 5 mm?

• b) What is the current density?
Solution

• a) The number of protons per second is:

• Here $n$ is the number of protons per second and $e$ is the charge of the proton.
Solution cont.

• b) The magnitude of the current density for this problem is just the current divided by the cross sectional area.
Drift Speed

- When a current is established in a circuit the electrons drift through the circuit with a speed that is related to the applied electric field.

- To determine the drift speed, imagine a section of wire of length L and cross sectional area A with number, n equal to the number of electrons per volume.
Drift Speed

• If the electrons all have the same speed then the time for them to move across the length $L$ of the wire is:
Drift Speed

• The current is then:
Drift Velocity

- The magnitude of the drift velocity can now be expressed as:

Then the current density is:
Charge Carrier Motion in a Conductor

- The zigzag black line represents the motion of a charge carrier in a conductor
  - The net drift speed is small
- The sharp changes in direction are due to collisions
- The net motion of electrons is opposite the direction of the electric field
Example Nerve Conduction

• Suppose a large nerve fiber running to a muscle in the leg has a diameter of 0.25 mm.
• When the current in the nerve is 0.05 mA, the drift velocity is $2.0 \times 10^{-6}$ m/s.
• If we model this problem by assuming free electrons are the charge carriers, what is the density of the free electrons in the nerve fiber?
Solution

• We first calculate the cross-sectional area of the nerve fiber.

• The current density is then:
Solution cont.

- We can now calculate the density of the free electrons.
Resistance

- The resistance of a circuit is defined as the potential drop across the circuit divided by the current that pass through the circuit.
- The unit for resistance is the ohm $\Omega = 1V/A$. 
Resistivity

• The resistivity of a material is defined as:

• The unit for resistivity is the ohm-meter.

• The resistance is a property of the entire object while the resistivity is a property of the material with which the object is made.
Resistance

• The relationship between resistance and resistivity is:
Resistivity and Conductivity

• The electric field can now be written in terms of the current and resistivity of the circuit.
• The conductivity of a material is the reciprocal of the resistivity.
Ohm’s Law

• Ohm's law states that the current through a device is directly proportional to the potential difference applied to the device.

• Note: Not all circuits obey Ohm's law.
• If the resistance is a function of the applied potential difference then the circuit will not obey Ohm's law.
Ohm’s Law cont.

- Ohm's law can be expressed by the following vector equation:

- An equivalent scalar equation for Ohm's law is given by:
Power in Electric Circuits

• By definition power is given as:

• Here \( P \) is power and \( U \) is the potential energy.

• The electric potential energy is given by:
Power in Electric Circuits

• We can now obtain the power of a circuit by differentiating the energy with respect to time.
Power in Electric Circuits

• If the potential difference is a constant with the time then the power can be expressed as:
Other Forms of Power

- If we use Ohm’s Law we can express the power as:

- The power of the circuit is the power dissipated by the resistance of the circuit.
Example

• Nikita, one of Section One’s top operatives, finds herself in a life-threatening situation. Red Cell has captured her and placed her in a containment cell with a large steel, electric locking, door. Nikita’s only chance to escape is to short-circuit the switch on the door from the inside.
Example cont.

- The switch has a fuse that will blow once the current exceeds 5.0 amps for more than 1.5s.
- Nikita has smuggled a small electrical device, given to her by Walter, into the cell.
- The device has a power rating of 25 W.
Example cont.

• a) What must the voltage of the device be in order to short-out the lock on the door?

• b) If the device has 50 J of energy stored in it, can Nikita open the door with this device?
Solution part a

• a) We can use the power equation to determine the minimum voltage needed to blow the fuse.
Solution part b

• b) The energy needed to blow the fuse can be determine by the following:
Resistance as a Function of Temperature

- We can express the temperature dependence of resistance in terms of the temperature coefficient of resistivity.
Resistance and Temperature

• We can solve this linear-first-order ordinary differential equation by using separation of variables method.
Resistance and Temperature

• If we integrate and solve for the resistivity we get the resistivity as a function of temperature.
• Note: as the temperature increase so does the resistivity.