Chapter 30

Inductance
Self Inductance

• When a time dependent current passes through a coil, a changing magnetic flux is produced inside the coil and this in turn induces an emf in that same coil.

• This induced emf opposes the change in flux.
Some Terminology

- Use \textit{emf} and \textit{current} when they are caused by batteries or other sources.
- Use \textit{induced emf} and \textit{induced current} when they are caused by changing magnetic fields.
- When dealing with problems in electromagnetism, it is important to distinguish between the two situations.
Self Inductance cont.

- The magnetic flux $\Phi_B$ passing through the $N$ turns of a coil is proportional to the current $I$ in the coil. Therefore, we define the self-inductance, $L$ as:
Self Inductance cont.

- The emf induced in a coil of self-inductance L can be obtained from Faraday’s law:
Self-Inductance, Coil Example

- A current in the coil produces a magnetic field directed toward the left (a)
- If the current increases, the increasing flux creates an induced emf of the polarity shown (b)
- The polarity of the induced emf reverses if the current decreases (c)
Example

• Calculate the value of the inductance for a tightly wrapped solenoid that has a length of 5.0 cm and a cross-sectional area of 0.30 cm² if the number of turns of the coil is 100.
Solution

- The inductance is given by the following:
Solution cont.

- We are given the number of turns but not the current or the flux; therefore, we need to determine the flux through the coil.
- The magnetic field of a solenoid is given as:
Solution cont.

• In the previous equation \( n \) represents the number of turns per unit length and \( \ell \) is the length of the solenoid.

• The flux is equal to the field times the cross-sectional area:
Solution cont.

- If we now substitute this expression into our equation for the inductance it becomes:
Solution cont.

• The inductance for the coil is then:
Inductance Units

- The SI unit of inductance is the **henry** (H)

\[ 1 \text{H} = 1 \frac{\text{V} \cdot \text{s}}{\text{A}} \]

- Named for Joseph Henry (pictured here)
Energy Stored in an Inductor

• An inductor is a device that can be used to store energy.
• If an inductor is placed in a circuit and a continually changing voltage is applied then an induced emf will be created inside the inductor.
Lenz’s Law

• According to Lenz’s law the polarity of the induced emf is opposite to that of the generator voltage.
• Therefore, the generator must perform work on the charges to push them through the inductor against the induced emf.
Energy of an Inductor

- The work done to push a small amount of charge through the inductor can be expressed as:
Energy of an Inductor

- If we remember that the current is the time rate of change of the charge then we can the following:
Energy of an Inductor

- After integration we get a relationship for the work in terms of the current in the inductor.
Energy of an Inductor

• By the work energy theorem we get:
Energy in a Solenoid

- If our inductor is a long solenoid then we can express the energy stored as:
Energy of a Solenoid

• The magnetic field for a solenoid is given as:

• Therefore, the energy stored by a current in a long solenoid is:
Energy Density of a Solenoid

- The energy density of the solenoid can be defined as the energy stored per unit volume and has the following form:
Example

- A coil of length 0.50-m and 200 turns of wire per meter is used to store enough energy to run your 100 W per channel stereo receiver at full volume for 20 minutes.

- How much current must be supplied to the coil to accomplish this if the coil has a diameter of 0.07 m?
Solution

• The energy needed to run the receiver is equal to the product of the power consumed by the receiver and the time at which it is running.
• The energy required to run a 100 W two channel stereo receiver for 20 minutes is:
Solution cont.

- The energy of a coil can be expressed as:
Solution cont.

• The current is then:
Example: The Coaxial Cable

- Calculate $L$ for the cable.

- The total flux is

$$
\Phi_B = \int B \, dA = \int_a^b \frac{\mu_0 I}{2\pi r} \, dr = \frac{\mu_0 I \ell}{2\pi} \ln\left(\frac{b}{a}\right)
$$

- Therefore, $L$ is

$$
L = \frac{\Phi_B}{I} = \frac{\mu_0 \ell}{2\pi} \ln\left(\frac{b}{a}\right)
$$

- The total energy is

$$
U = \frac{1}{2} L I^2 = \frac{\mu_0 \ell I^2}{4\pi} \ln\left(\frac{b}{a}\right)
$$
Mutual Inductance

- If two coils are placed next to each other and one coil has a changing current passing through it the second coil will experience a changing magnetic flux through it.
- This changing flux will produce an induced emf in the second coil.
Mutual Inductance, 2

- The current in coil 1 sets up a magnetic field
- Some of the magnetic field lines pass through coil 2
- Coil 1 has a current $I_1$ and $N_1$ turns
- Coil 2 has $N_2$ turns
Mutual Inductance cont.

- The relationship for mutual inductance is as follows:
Mutual Inductance cont.

• In the previous equation $M$ represents the mutual inductance, $N_2$ is the number of turns in the second coil, $\Phi_2$ is the flux passing through the second coil, and $I_1$ is the current in the first coil.

• The units for inductance are called the Henry $H$. 
Mutual Inductance cont.

- If we apply Faraday’s law we can derive a relationship between the mutual inductance and the emf created in the second coil by the first.
Example

• A long thin solenoid of length $L$ and cross-sectional area $A$ contains $N_1$ closely packed turns of wire.

• Wrapped around it is an insulated coil of $N_2$ turns.

• Assume all the flux from coil 1 (the solenoid) passes through coil 2, and calculate the mutual inductance.
Solution

• We first determine the flux produced by the solenoid.

• The magnetic field inside the solenoid is given by the following:
Solution cont.

• The flux induced in the second coil due to the first is then:
Solution cont.

- Hence the mutual inductance is:
The RLC Circuit

- Consider a circuit with an inductor, a charged capacitor, and a resistor all hooked together in series.
- The energy stored in the capacitor will begin to flow over into the inductor.
- Meanwhile, the current in the circuit will be dissipated by the resistor.
The RCL Circuit cont.

• The equation describing this can be written as:
The RCL Circuit cont.

• If we now divide the equation by the current and then use the relation that the current is the time derivative of the charge then we get the following:
The RCL Circuit cont.

- This is an ordinary second order differential equation with constant coefficients.
- It has solutions that look like the following:
The RCL Circuit cont.

- The angular frequency in the previous equation is the circuit’s damped oscillating frequency and is given by:
The RCL Circuit cont.

• Note: in the absence of a resistor, the solution to the differential equation is that of a harmonic oscillator with an angular frequency of: