Quantum Teleportation Pt. 1

PHYS 500 - Southern Illinois University

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Types of Communication

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Let us look at more general communication scenarios.
Classical and Quantum Channels

A classical channel is any medium that can transmit classical information from one point in spacetime to another, such as the telephone, email, zellephone, etc.

Physically, a quantum channel is some medium that is used to transmit the state of a quantum system from one point in spacetime to another, such as a fiber optic cable, optical cavities, etc.

Mathematically, quantum channels are represented by completely positive trace-preserving (CPTP) maps.

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Quantum Communication Models

Important Types of Models

1. Classical Communication over a Classical Channel
2. Classical Communication over a Quantum Channel
3. Quantum Communication over a Quantum Channel
4. Classical Communication over an Entanglement-Assisted Quantum Channel
5. Quantum Communication over an Entanglement-Assisted Classical Channel
Classical Communication over a Classical Channel

The Picture
Classical Communication over a Quantum Channel

The Picture
Quantum Communication over a Quantum Channel

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The Picture

Schumacher compression involves the compression of some quantum source and transmission over a quantum channel.
Quantum Communication over a Quantum Channel

A quantum channel allows for entanglement distribution from Alice to Bob.
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Classical Communication over an Entanglement-Assisted Quantum Channel

The Picture
Superdense Coding

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Classical Communication over an Entanglement-Assisted Quantum Channel

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Suppose that Alice and Bob share one of the Bell states

\[ |\Phi_{00}\rangle = \sqrt{1/2}(|00\rangle + |11\rangle). \]

Recall that the other Bell states are obtained by a local unitary applied on Alice’s side only

\[ |\Phi_{b_1b_2}\rangle = \sigma_z^{b_1} \sigma_x^{b_2} \otimes I |\Phi_{00}\rangle \quad (b_1, b_2) \in \{0, 1\}^2. \]
Alice then has the ability to send Bob 2 bits of classical information using their shared Bell state and the quantum channel.
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She first encodes:

$$(b_1, b_2) \in \{0, 1\}^\times 2 : |\Phi_{00}\rangle^{AB} \rightarrow |\Phi_{b_1 b_2}\rangle^{AB} = \sigma_z^{b_1} \sigma_x^{b_2} \otimes I |\Phi_{00}\rangle^{AB}.$$
Classical Communication over an Entanglement-Assisted Quantum Channel

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She then sends her qubit in system \(A\) (which is entangled with \(B\)) over the quantum channel into Bob’s lab. Bob relabels the incoming system as \(B'\):

\[|\Phi_{b_1b_2}\rangle^{AB} \rightarrow |\Phi_{b_1b_2}\rangle^{B'B}.\]
Classical Communication over an Entanglement-Assisted Quantum Channel

Bob is then able to identify which of the four Bell states he holds by measuring systems $B'B$ in the Bell basis:

$$|\Phi_{00}\rangle^{B'B} = \sqrt{1/2}(|00\rangle + |11\rangle)$$
$$|\Phi_{01}\rangle^{B'B} = \sqrt{1/2}(|01\rangle + |10\rangle)$$
$$|\Phi_{10}\rangle^{B'B} = \sqrt{1/2}(|00\rangle - |11\rangle)$$
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It is called “superdense coding” because with shared entanglement, Alice is able to send two bits of classical information using just one qubit of quantum information.
Quantum Communication over an Entanglement-Assisted Classical Channel

The Picture