Quantum Teleportation Pt. 3

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A Bit of History on Teleportation

The original teleportation paper was published in 1993 and according to Google Scholars has 11,371 citations.

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Teleporting an Unknown Quantum State via Dual Classical and Einstein-Podolsky-Rosen Channels

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To put this in perspective, Einstein's most frequently cited paper has a citation count of 15,209 (which, incidentally, is also a quantum information paper - the "EPR paper").

A Bit of History on Teleportation

How was the teleportation protocol discovered?

This is the story told to me by Gilles Brassard (one of the authors on the teleportation paper):

The development of the teleportation protocol has its roots in a 1991 paper by Asher Peres and Bill Wootters.

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Optimal Detection of Quantum Information

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Optimal Detection of Quantum Information

The Peres-Wootters paper investigates the following problem.

Suppose that Alice and Bob are each given the same qubit state, and they are promised that the state is one of three possibilities (but they don't know which one).

Possible states:

$$|\psi_0\rangle = |0\rangle, \qquad |\psi_1\rangle = \tfrac{1}{2}|0\rangle + \sqrt{\tfrac{3}{2}}|1\rangle, \qquad |\psi_2\rangle = \tfrac{1}{2}|0\rangle - \sqrt{\tfrac{3}{2}}|1\rangle.$$

The set $\{|\psi_0\rangle, |\psi_1\rangle, |\psi_2\rangle\}$ is known as the **trine ensemble**.

So Alice and Bob's joint state is one of the following

$$|\Psi_0\rangle^{AB} = |\psi_0\rangle^A|\psi_0\rangle^B, \quad |\Psi_1\rangle^{AB} = |\psi_1\rangle^A|\psi_1\rangle^B, \quad |\Psi_2\rangle^{AB} = |\psi_2\rangle^A|\psi_2\rangle^B.$$

Optimal Detection of Quantum Information

To make their guess, Alice and Bob are allowed to perform local quantum measurements and discuss their measurement outcomes with one another, a process known as **local quantum operations and classical communication** (LOCC).

In the end, they use their collective measurement outcomes and agree on a guess for their state's identity.

What is the optimal strategy for guessing the identity of the given state

$$|\Psi_0\rangle^{AB} = |\psi_0\rangle^{A}|\psi_0\rangle^{B}, \quad |\Psi_1\rangle^{AB} = |\psi_1\rangle^{A}|\psi_1\rangle^{B}, \quad |\Psi_2\rangle^{AB} = |\psi_2\rangle^{A}|\psi_2\rangle^{B}?$$

Do LOCC measurements render a smaller success probability compared to global measurements (i.e. general measurements across full state space AB)? Intuitively they should not since the $|\Psi_i\rangle$ are not entangled.

Optimal Detection of Quantum Information

Findings of the Peres-Wootter paper suggest the following conclusions:

1. Multiple rounds of interactive classical communication can improve the LOCC success probability.
2. Even the best LOCC measurement cannot match best global success probability.

But conclusion 2 was not fully proven. The question remained open whether or not some complicated LOCC measurement could, in fact, also achieve the global optimal success probability.

In 1992, the authors of the teleportation paper in Montreal to discuss this open problem. Charlie Bennett asked the question of what would happen if Alice and Bob had some additional shared entanglement. When reflecting upon that question, the group discovered the teleportation protocol.

Superluminal Communication in Teleportation?

Return to the standard teleportation protocol $\mathcal{T}_0$ on the Bell state $|\Phi^+\rangle$. The protocol will succeed regardless of how far apart Alice and Bob are from one another. Does teleportation allow for faster-than-light (i.e. superluminal) communication?

Teleportation involves sending quantum information (i.e. states of a quantum system) from Alice to Bob. Does she do so superluminally?

The answer is no. The initial state of Alice and Bob's system is $|\eta\rangle^{A'}|\Phi^+\rangle^{AB}$. Bob describes his system as the density operator

$$\rho_1^B = tr_{A'A}\left(|\eta\rangle\langle\eta|^{A'} \otimes |\Phi^+\rangle\langle\Phi^+|^{AB}\right) = \frac{1}{2}\mathbb{I}.$$

Superluminal Communication in Teleportation?

After Alice makes her Bell measurement but *before Bob learns the outcome*, Bob describes his system as the ensemble average

$$\rho_2^B = tr_{A'A}\left(\sum_{b_0,b_1=0}^{1} |\Phi_{b_0 b_1}\rangle\langle\Phi_{b_0 b_1}|^{A'A} \otimes \sigma_z^{b_0}\sigma_x^{b_1}|\eta\rangle\langle\eta|\sigma_z^{b_0}\sigma_x^{b_1}\right)$$

$$= \sum_{b_0,b_1=0}^{1} \sigma_z^{b_0}\sigma_x^{b_1}|\eta\rangle\langle\eta|\sigma_z^{b_0}\sigma_x^{b_1} = \frac{1}{2}\mathbb{I}.$$

Hence $\rho_1^B = \rho_2^B$. In other words, Alice's measurement alone is not able to affect the description of Bob's system.

Only *after* Bob learns Alice's outcome can he perform error correction and obtain the new density operator $\rho_3^B = |\eta\rangle\langle\eta|$.

Combine Teleportation with Superdense Coding

Recall the resource trade-off established by teleportation: $[ebit] + 2[\overrightarrow{CC}] \geq [\overrightarrow{QQ}]$. Do we need 2 bits of CC to teleport 1 qubit?

No superluminal communication requires that 2 bits are needed for teleportation. This can be seen by combining teleportation with superdense coding:

If, say, 1 bit of CC were sufficient for teleportation, then Alice could instantaneously send Bob a 2-bit message with error rate 50%, which is less than the error rate of just guessing (75%). Channel coding would allow for reliable superluminal communication.