Dear Dr. Bukman

Attached please find our manuscript, EE10362 "Normal modes in model jammed systems," by Silbert, et al. We thank the referee for a perceptive and thorough review of the paper and the thoughtful comments that will help us clarify our meaning in several places.

We have revised the manuscript according to the comments of the referee and are resubmitting it for publication. Our detailed response is given below. We show the referee's comments in boldface and our responses follow immediately. At the end of this letter we list the changes made to manuscript.

Yours sincerely

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Report of the Referee -- EE10362/Silbert

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This paper shows an analysis of the vibrational spectra and normal modes in a 2D assembly of monodisperse harmonic disks, in a 3D assembly of randomly positioned monodisperse soft-spheres interacting through harmonic and/or Hertzian potential. The paper is well written and interesting, since it provides a first extensive study of the evolution of normal modes as a function of the packing fraction, especially close to the jamming transition.

We agree with the referee that our work is interesting and that the manuscript provides an extensive study of the normal modes in jammed packings.

However, the presentation that mixes 2D and 3D results is confusing, and the conclusion, in its present form, is far from been very convincing. I would thus recommend this paper for publication after the authors have provided some changes in their manuscript. The following points have especially to be clarified:

1. The type of potential used (harmonic or hertzian for 3D systems) is never discussed. Do the author noticed any difference between them?

We thank the referee for pointing out an inconsistency in the text. All of the results shown are for the harmonic potential. We have clarified this in the text on page 1.

2. The system (2D or 3D) is not clearly noticed in fig. 4-7-8-9-10. The fact that some figures refer to 2D systems (fig.2) and others to 3D systems is confusing. It is shown in fig.6 that both 2D or 3D systems are very different. The authors should better do a systematic comparison between them.

We have now specified the dimensionality in each figure caption. To focus the paper and clarify our results, we have now removed most of the 2D data. All the analyses that we report here have been carried out only in 3D. The only 2D results left in the manuscript are the images of the modes in Fig. 2. It is now clearly stated that this figure is there only to give the reader a visualization of what the modes look like in the different regimes. (It would not be possible to do this effectively in 3D.)

3. The size of the 2D systems is not clearly mentioned in the introduction, nor in the corresponding figures.

The system size (N=10000) is stated in the introduction and in the figure caption of Fig.2.

4. The choice of a different $\Delta\Phi$ in order to show the regime B in figure 2 is also confusing. It is not clear wether the initial plateau shown in systems close to jamming corresponds to the plateau shown in the density of vibrational states far from jamming!

This is an excellent point and we have studied this issue further. We have now made a distinction between modes from the plateau region at very low frequencies and those at intermediate frequencies. These are now labeled B' and B respectively in Fig. 1(a), (b). We have also added an image to Fig.2 that distinguishes modes in regions B' and B. These images show that the modes look similar but contain subtle differences.

We explore this distinction further with a new figure included in the revised manuscript. This appears as Fig.7 and related discussion on page 4. We plot a characteristic length scale that we obtain from the correlation functions shown in Fig.6. This length scale characterizes the extended nature of the modes. It decreases with increasing frequency and, interestingly, data for different $\Delta \phi$ converge at values of the frequency where the plateau regions in the density of states also coincide. These data are consistent with the picture that the modes in the plateau region are similar and only differ by how extended they are. We explicitly state all of this in the text in the revised manuscript.

5. The "boson peak" generally refer to a cross over between a w^(D-1) behavior and a w^a behavior in the density of vibrational states. Usually a\neq 0 (cf. double well potential models). If a=0, it should be discussed more extensively. The boson peak is not systematically related with the place where "the density of vibrational states becomes approximately flat" (p.2 and in the conclusion).

The referee is correct that the nomenclature about the "boson peak" should be clarified. Because the phrase "boson peak" is used rather loosely in the literature, we have modified the manuscript to avoid making specific mention of it. We now leave out any reference to the boson peak except in the introduction. When we talk about the excess modes we now refer to them precisely. (In our original manuscript, by "boson peak" we meant simply where the excess low-frequency modes are located. Of course, these modes are spread out over a range of frequencies; the density of states in our 3D system behaves roughly as w^1 at low frequencies, crossing over to w^0 at higher frequencies. In previous work, we used a common definition of the boson peak, which is the maximum of $D(\omega)/\omega^2$, and showed that the peak frequency shifts systematically to zero and the maximum diverges as $\Delta \phi \rightarrow 0$ (Ref. [8]).)

6. In order to emphasize the departure from plane waves, the author should study the normal modes, after having substracted the contribution of the main projection on plane waves. In fig. 6 for example, it is not clear wether the evolution of C(r) is due to plane wave, or to the "non-affine" field. It is also necessary if they want to compare their result with ref. 18. Same remark for the participation ratio fig. 3, and for the fourier transforms fL-k,w) and fT(k,w).

This would be an interesting analysis, but it is beyond the scope of this paper. The message of the extensive calculations that we have described is that differences between modes in Regimes A and B are small compared to the change of the density of states with compression, and that there are only subtle changes in the behavior of the modes as a function of packing fraction. This is an important and interesting point. We have now stated this point explicitly in the conclusions. The structure factors f_L and f_T show clearly that the modes are quite different from plane waves and have significant contributions from a broad range of wavevectors. Our aim here is not to emphasize the departure from plane waves but to take a look at the nature of the modes as a function of packing fraction and frequency.

7. A large part of the paper is devoted to the analysis of the fourier transforms fL(k,w) and fT(k,w). However, ther is no marked difference in it for different packing fraction. Do this result contredicts the conclusion ?

We thank the referee for pointing this out. We have rewritten the conclusion to make our result more clear and have now removed the first sentence of the former conclusion.

8. It would be interesting to provide a more detailed study of the evolution of the position of the peaks in the dynamical structure factor with \omega. What comment would do the authors about the corresponding wavelength?

We have some limited studies that show that the peak position shifts as ω/c, where c is the corresponding sound velocity.

9. Finally, the authors should avoid to mention in the conclusion that "clearly more work needs to be done to explore the nature of these modes in detail", because it is precisely the main interest of this article to provide a detailed study of these modes... The author should point out more clearly their conclusion about the nature (localized or not, for example) of these modes. They could also relate their results to other numerical studies, and discuss the differences between 2D and 3D systems.

We have rewritten the summary, specifying more clearly what our conclusions are.

List of primary changes

1. Revised parts of text in the introduction section and Eq.1. Added a new reference, [4], in the introduction section.
2. A regime B' in Fig. 2(a) has been introduced. The text related to Fig. 2 has also been revised.
3. We have added a two panels to Fig. 3, and updated the caption to Fig. 3 accordingly.
4. Revised Fig. 6. There are now two panels to Fig. 6, corresponding to different frequency regimes for different values of $\Delta\phi$.
5. Added a new figure, Fig. 7, containing our new analysis of the characteristic length scale associated with mode correlations. A new paragraph related to the data analysis of Fig. 7 has also been included.
6. In light of recent developments, we now reference unpublished work, [20], which is related to some of the data presented in our manuscript.
7. Revised parts of the text related to Figs. 8, 9, and 10.
8. Clarified the summary and conclusions portion of the text.